

# Synergies with the Cosmic Microwave Background Radiation.

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## Abstract

The Cosmic Microwave Background (CMB), just like the HI 21 cm radiation, constitutes a privileged witness of the evolution of our universe during and after the epoch of reionization. This parallelism between the two radiation fields is also reflected in the fact that cosmological events leave different (but often complementary) signatures on both photon fields. In fact, there exists a number of physical channels of interaction between the CMB radiation with electrons, atoms, ions and gravitational potentials during and after the epoch of reionization: these strongly motivate a joint study of the CMB and future HI 21 cm data. In this context, we also highlight the existing parallelism between problems addressed in CMB analysis and problems foreseen in future HI 21 cm observations, in many of which the Spanish cosmological community has been deeply involved during the last decade.

## 1 Introduction

Just like the CMB photons, the HI 21 cm photons, emitted by neutral hydrogen some time between cosmological recombination and cosmological reionization, reach us after crossing a very significant volume of the observable universe. Both the CMB and HI 21 cm photon fields witness an evolving universe that becomes ionized by second time while the first stars are born, structure formation proceeds and metals are synthesized in stars and ejected to the Intergalactic Medium (IGM). All these physical processes leave their signature in the CMB and HI 21 cm photon fields in different (and often complementary) ways, and just for this reason a joint analysis in both radio and millimeter wavelengths is extremely interesting from

the cosmological point of view. Furthermore, the evolution of the HI 21 cm field is linked to the evolution of the CMB photon bath itself, since the population of the level involved in the HI 21 cm transition is heavily influenced by the CMB. Depending on the dominant physical processes, the HI 21 cm line may be seen in emission or in absorption with respect to the CMB field, which acts as a background light.

In this chapter we describe those secondary anisotropies induced in the CMB during and after the epoch of reionization which can benefit from the combination with radio observations of the (redshifted) HI 21 cm line. We first address the interaction of CMB photons with metals and ions synthesized during the epoch of reionization, and discuss how this interaction may provide new insight on the physics of reionization. We next revisit the interaction of the CMB with free electrons in the IGM via inverse Compton scattering, and study its degree of complementarity with HI 21 cm observations. We also study how a deep galaxy survey built upon HI observations may also help to unveil the impact of the universal acceleration on the CMB by means of the integrated Sachs-Wolfe effect (ISW). Finally, we conclude by highlighting the similarity existing between the problems to be addressed in the analysis of HI 21 cm data and those related to CMB analysis, in which the Spanish cosmological community has extensively worked in the last decade.

## 2 Interaction of the CMB with atoms, molecules and plasma during the epoch of reionization

In this section we briefly describe the physics of the interaction of CMB photons with fine structure transitions associated to atomic, ionic and molecular species present in the IGM. Such species should be the result of the pollution induced by the explosion as supernovae of Population III stars, and hence these phenomena may allow to trace the chemical enrichment of the IGM at the redshifts of interest. We also make brief reference to those other processes injecting energy in the CMB and causing spectral distortions accessible by the next generation of CMB missions.

### 2.1 Resonant scattering on fine structure transitions during reionisation

It was shown in Basu et al. (2004) (hereafter B04) that a CMB photon presently observed at frequency  $\nu_{\text{obs}}$  may have interacted with a resonant transition  $X$  of resonant frequency  $\nu_X$  at a resonant redshift  $z_X = \nu_X/\nu_{\text{obs}} - 1$ . For the range of frequencies for which the CMB can be clearly observed (say 10–300 GHz), transitions in the fine structure range (20–600  $\mu\text{m}$ ) will fall naturally in a redshift range bracketing the reionization epoch ( $z_X \in [5, 50]$ ). Since in resonant scattering the absorbed CMB photon is simply re-emitted in a different direction, the number of CMB photons is conserved, there is no distortion of the CMB black body spectrum and this process can only be seen in the angular anisotropies of the CMB.

B04 showed that the leading impact of this process on the CMB angular power spectrum is a frequency dependent *blurring* of the intrinsic CMB anisotropies generated at  $z \simeq 1, 100$ . If a given species is generating an optical depth  $\tau_X$  associated to a resonant transition  $X$ ,

then the ratio of the power spectrum of two CMB maps covering the same patch of the sky, but observed at two different frequencies  $\nu_1$  and  $\nu_2$ , will equal

$$C_{l,1}/C_{l,2} = \frac{\exp -2\tau_{X,1}}{\exp -2\tau_{X,2}} \simeq \frac{1 - 2\tau_{X,1}}{1 - 2\tau_{X,2}}. \quad (1)$$

The optical depths  $\tau_{X,1}$ ,  $\tau_{X,2}$  are both associated to the same resonant transition  $X$  but observed at different resonant redshifts  $z_{X,i} = \nu_X/\nu_i - 1$ , with  $i = 1, 2$ . We are assuming that both are much smaller than unity, as it is in practically all cases. The optical depth  $\tau_{X,2}$  is proportional to the oscillator strength of the resonant transition  $X$  and the *cosmic average* number density of resonant atoms/ions at redshift  $z_{X,2}$ . If  $\nu_2$  is chosen so that  $z_{X,2}$  is high enough to render the number of resonant atoms/ions negligible, then  $\tau_{X,2} \simeq 0$  and from Eq. 1 above B04 find that

$$C_{l,1} \simeq C_l^{\text{CMB}} (1 - 2\tau_{X,1}), \quad (2)$$

where the term  $C_l^{\text{CMB}}$  refers to the intrinsic CMB angular anisotropies generated during cosmological recombination. B04 concluded that by comparing CMB maps at different frequencies it may be possible to set constraints on the abundance of a resonant species at redshifts  $z_{X,1}$ . They also identified a number of resonant transitions (CI 370.4  $\mu\text{m}$ , CII 157.7  $\mu\text{m}$ , OI 63.2  $\mu\text{m}$ , OIII 88.4  $\mu\text{m}$ , etc) whose effect should be dominant and at worst close to the limit of detectability in future CMB missions. This pattern described so far is the predicted impact of the *average* number density of resonant atoms/ions at the resonant redshift  $z_{X,1}$ . The patchiness in the metal distribution will reflect in further anisotropies on angular scales comparable to the typical clustering of the bubbles containing the resonant species, but typical at much smaller amplitudes that will be neglected hereafter.

Given the small amplitude expected for realistic values of cosmic abundances of resonant species (the  $\tau_X$ 's are typically below  $10^{-3}$ ), the level of foreground removal and control of systematics is highly demanding (see Hernández-Monteagudo et al. (2006) for details). Nevertheless, *a priori* this effect should show up in both intensity and polarization CMB anisotropies (Hernández-Monteagudo et al., 2007), and should lie within the range of detectability of foreseen future CMB missions of the type of PRISM<sup>1</sup>.

The cross-correlation of HI 21 cm maps with future, high sensitivity CMB maps in both intensity and polarization would provide an unprecedented view of the history of cosmic enrichment of the IGM during the end of the Dark Ages and the epoch of reionization. The HI 21 cm observations will perform a tomographic study of distribution of HI during reionization. Similarly, accurate multifrequency CMB observations will provide a tomographic description of the pollution of the IGM with metals and ions during the same epoch. These metals and ions will be expelled by supernovae and thus are likely to follow the same spatial pattern of HII. Thus, *a priori*, it is expected that the blurring of CMB anisotropies induced by metals will be spatially anti-correlated to HI 21 cm emission, although an actual comparison or cross-correlation study will shed light on how metal pollution versus hydrogen ionization proceeded during reionization. Provided that systematics from radio and CMB observations are expected to be of very different nature, we can expect that this comparison would consti-

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<sup>1</sup>PRISM's URL site: <http://www.prism-mission.org>

tute a robust consistency test on the cosmological constraints derived for reionization from these two data sets.

## 2.2 Distortions induced in the CMB black body spectrum during reionization

Although the CMB is, to date, the most perfect black body that has ever been measured, increasing interest is arising around the study of deviations from this black body law. Possible future missions like PRISM or PIXIE (Kogut et al., 2014) should be sensitive to levels of spectral distortions of the CMB about 100–1000 times smaller than current upper limits. The presence of a re-ionized medium must introduce a floor distortion of the CMB spectrum at the level of  $10^{-8}$ – $10^{-7}$ . But, addition to this, it is expected that other physical processes inject energy to the CMB photon bath, which are potentially detectable by the experiments just quoted above. Just to cite a few examples occurring during the epoch of reionization, such observations would shed light on the amount of energy released by shocks and the first supernovae during reionization (Cen & Ostriker, 2006; Miniati et al., 2000; Oh et al., 2003); they would also set constraints on the impact of non-linear bulk flows on the spectrum of CMB polarization (Renaux-Petel et al., 2014); and they may even detect how the first stars pollute the IGM with oxygen and distort the CMB spectrum via the Wouthuysen-Field effect on the OI 63.2  $\mu\text{m}$  line (Hernández-Montenegro et al., 2007, 2008).

Again, these observations would provide an alternative view of the onset and development of reionization, independently of that given by the HI 21 cm line. Therefore a joint study of both HI 21 cm and CMB distortion maps (suffering each from different systematics) would provide hints about the dominant physical processes which, during reionization, transferred energy and metals to the IGM.

## 3 The kinetic and thermal Sunyaev-Zeldovich effects during the epoch of reionization

The kinetic and thermal Sunyaev-Zeldovich effects (hereafter kSZ and tSZ, respectively Sunyaev & Zeldovich, 1972, 1980) describe the inverse Compton scattering of CMB photons off free electrons in the intergalactic medium. If there is no energy transfer between the CMB photons and the electrons (Thomson scattering), then the black body spectrum of the CMB is preserved and only the Doppler kick due to the relative motion of the electrons with respect to the CMB rest frame may introduce a (local) dipolar modulation of the CMB brightness. This is the kSZ effect. If instead the electrons are hot, then there exists energy transfer into the CMB photon bath that effectively distorts the CMB black body spectrum (tSZ effect).

On its way from the last scattering surface to us, the CMB photons propagate through a universe that, right after recombination, is filled with neutral hydrogen and helium. These *dark ages* come to an end by  $\sim 30$ – $50$  as the first stars are formed (cosmic dawn), and the UV radiation emitted by these ionizes the intergalactic medium in bubbles that eventually percolate, leaving a newly ionized universe. How this *epoch of reionization* (EoR) proceeds

is practically unknown, but both the tSZ and the kSZ effects can provide useful insight. Predictions about the tSZ are more uncertain, since the tSZ is proportional to the electron pressure which is highly dependent on the non-linear physics dominant during the EoR (but see Oh et al., 2003, and references to this work), whereas the kSZ depends solely on the electron density and velocity and thus predictions are less dependent of non-linear physics, see, e.g., Hernández-Monteagudo & Ho (2009) for a second order estimation of the all sky kSZ power spectrum and Zahn et al. (2005); Trac & Cen (2007) for studies based upon hydrodynamical numerical simulations. These studies showed that the kSZ would dominate over the tSZ during the EoR, and that the amplitude of the kSZ would depend on the duration and degree of patchiness of the re-ionization process. When the kSZ is not referred to electrons in collapsed structures, but rather to large scale, smooth electron distributions, it is also called the Ostriker-Vishniac (OV) effect.

At the same time, the EoR leaves also a trace in the radio part of the spectrum where the 21-cm line vanishes due to the absence of neutral hydrogen. SKA will probe this regime directly and be able to map in redshift slices the distribution of neutral hydrogen. Observations made with SKA will be crucial to understand the role of the kSZ effect in future CMB experiments. The cross-correlation of SKA data with future high-sensitivity, high-resolution stage IV CMB experiments (Abazajian et al., 2015), will help boost the signal from the EoR facilitating its study in greater detail (Slosar et al., 2007). In particular, Jelic et al. (2015) show how the cross-correlation between the cosmological 21 cm line and the kSZ is maximized for patchy reionization and peaks at scales  $l \approx 100$  (correlation) and  $l \approx 5000$  (anticorrelation). The positive correlation at large scales is due to the natural correlation of overdensities on large scales while the anticorrelation on small scales reflects the complementarity nature of the kinetic SZ and the 21 cm line. The same authors also consider the particular case of SKA and predicts a signal to noise of approximately 3 for an instantaneous reionization with 1 000 hours of integration in SKA and a *Planck*-like sensitivity for the kSZ (see also Tashiro et al., 2010).

The combination of kSZ and 21 cm line data from SKA opens the door also to novel analyses. Both the kSZ from the EoR and the 21 cm line signal from the same period can be used as a background for lensing studies. The combination of kinetic SZ and 21 cm line data can be used to increase the signal to noise of this background lensing plane at  $z \approx 10$ . Using this background for lensing studies has a variety of advantages that make it an attractive alternative for cosmological studies. This technique has been tested in the past with simulations Zahn & Zaldarriaga (2006); Diego & Herranz (2008); Metcalf & White (2009) with promising results. Similar techniques are nowadays starting to be applied but using the CMB as a background.

## 4 Combining CMB and HI 21 cm observations in the context of the Integrated Sachs Wolfe effect

The late integrated Sachs-Wolfe (ISW) effect (Sachs & Wolfe, 1967; Rees & Sciama, 1968; Martinez-Gonzalez et al., 1990) describes the gravitational interaction suffered by the CMB

photons when passing through the time-evolving LSS. This secondary anisotropy of the CMB is given by an integral of the evolution of the gravitational potential  $\Phi$ , as a function of the conformal time  $\eta$ :

$$\frac{\Delta T}{T_{\text{CMB}}} = -\frac{2}{c^3} \int_0^{\eta_{\text{CMB}}} d\eta \dot{\Phi}, \quad (3)$$

where  $\eta_{\text{CMB}}$  represents the conformal time at recombination, corresponding at a redshift of  $z \simeq 1100$  in a  $\Lambda$ CDM cosmology. The ISW effect is very difficult to detect from CMB measurements, first, because it is a very weak signal and, second, because the CMB distortion caused by the time evolution of the gravitational potential redshifts (or blueshifts) the primary CMB photons, conserving its black body electromagnetic spectrum. However, it is possible to detect this effect by cross-correlating a map of the CMB anisotropies with tracers of the LSS, as original proposed by (Crittenden & Turok, 1996).

Because the ISW effect depends on the growth of structures, it is useful to constrain physics behind such evolution: a cosmological constant (e.g., Nolta et al., 2004), dark energy (e.g., Vielva et al., 2006), modified gravity (e.g., Zhao et al., 2010), or non-flat curvature (e.g., Li & Xia, 2010).

Since its first detection in 2004, through the cross-correlation of CMB data from WMAP and X-ray (HEAO-1) and radio (NVSS) catalogues (Boughn & Crittenden, 2004), several works (e.g., Fosalba et al., 2003; Nolta et al., 2004; Vielva et al., 2006; Giannantonio & Melchiorri, 2006; Cabré et al., 2007; Ho et al., 2008; Planck Collaboration et al., 2014) have confirmed the detection of the ISW effect (with significance levels ranging from  $\approx 2\sigma$  to  $4\sigma$ ). Recently, the Planck Collaboration has reported a  $4\sigma$  detection, by the combination of NVSS, SDSS and WISE catalogues, and the *Planck* lensing map (Planck Collaboration et al., 2015).

For an ideal survey, i.e., full-sky, negligible shot noise, and mapping, at least, up to  $z \sim 4$ , the maximum signal-to-noise ratio for the detection of the ISW effect is  $\approx 8$ . Among the previous factors, the sky coverage is the most important limitation. Since SKA is expected to provide an almost half-sky coverage, it is possible to obtain a detection level  $\approx 4\sigma$ , similar to the one already obtained by the combination of multiple surveys, as mentioned above.

The possibilities that SKA offers to study the ISW effect are not limited to the signal detection, and the derived constraints on cosmological parameters. On the one hand, the precise redshift estimation of SKA galaxies by means of the HI 21 cm line will allow performing ISW-tomography, which can be very useful, in particular, to estimate a map of the ISW fluctuations, as function of redshift (using filtering techniques as the ones described in Barreiro et al., 2008). This can be used to probe further the nature of some of the CMB large-scale anomalies that puzzled the cosmological community (Planck Collaboration et al., 2014). On the other hand, the cosmic network that is mapped by the SKA galaxies will identify superstructures as clusters and voids, which can be used to prove the ISW effect through the stacking of the CMB fluctuation on the location of this structures. This kind of analyses has in fact revealed some anomalous signals, which are difficult to interpret in the context of the standard  $\Lambda$ CDM model (e.g., Hernández-Monteagudo & Smith, 2013; Ilić et al., 2013; Planck Collaboration et al., 2015).

## 5 Foreground removal: from CMB to 21 cm

The extraction of cosmological information from the analysis of the intensity mapping of the 21 cm requires of a precise process to disentangle the cosmological signal from foreground emission. According to its spatial properties and origin, these foregrounds can be classified as diffuse, anisotropic and galactic (as the synchrotron and free-free radiations) and compact, isotropic and extragalactic. The foreground emission is, in general, much higher than the cosmological 21 cm fluctuations and, therefore, its accurate removal is challenging.

The foreground removal in the 21 cm context has several similarities with the CMB component separation problem that has been widely studied during the last almost 20 years: maximum-entropy (MEM, e.g., Hobson et al., 1998; Barreiro et al., 2004), spectral matching (SMICA, e.g. Patanchon et al., 2005), internal linear combinations (Basak & Delabrouille, 2012; Fernández-Cobos et al., 2012), independent component analysis (Maino et al., 2002, FastICA, e.g.), correlated component analysis (CCA, e.g., Bedini et al., 2005) ... However, one relevant difference between the component separation problem in the CMB and the 21 cm frameworks is that, while for the former the cosmological signal is the same among different observational frequencies, the source emitting the 21 cm signal is different across the frequency range, due to cosmological redshift. It is also worth mentioning that, as for the CMB case, the wealth of information encoded within the 21 cm measurements aim not only to obtain the cosmological signal, but also to separate and characterise the different foreground components, thus enabling an improved understanding of the physical processes behind them.

Another significant difference between HI 21 cm and CMB is that, while for the CMB case all signals (foregrounds and primordial) are highly smooth with respect to observational frequency, for the 21 cm the cosmic signal associated to neutral hydrogen exhibits much more structure. Despite of being a much younger field than the CMB, the study of the component separation problem in the context of 21 cm observations has already imported methods and techniques used in CMB analyses, (e.g., Ansari et al., 2012; Chapman et al., 2012, 2015; Alonso et al., 2015).

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