

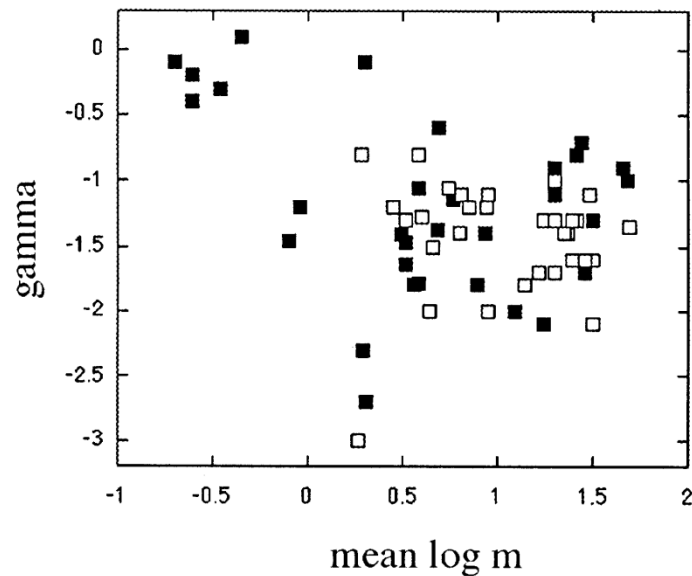
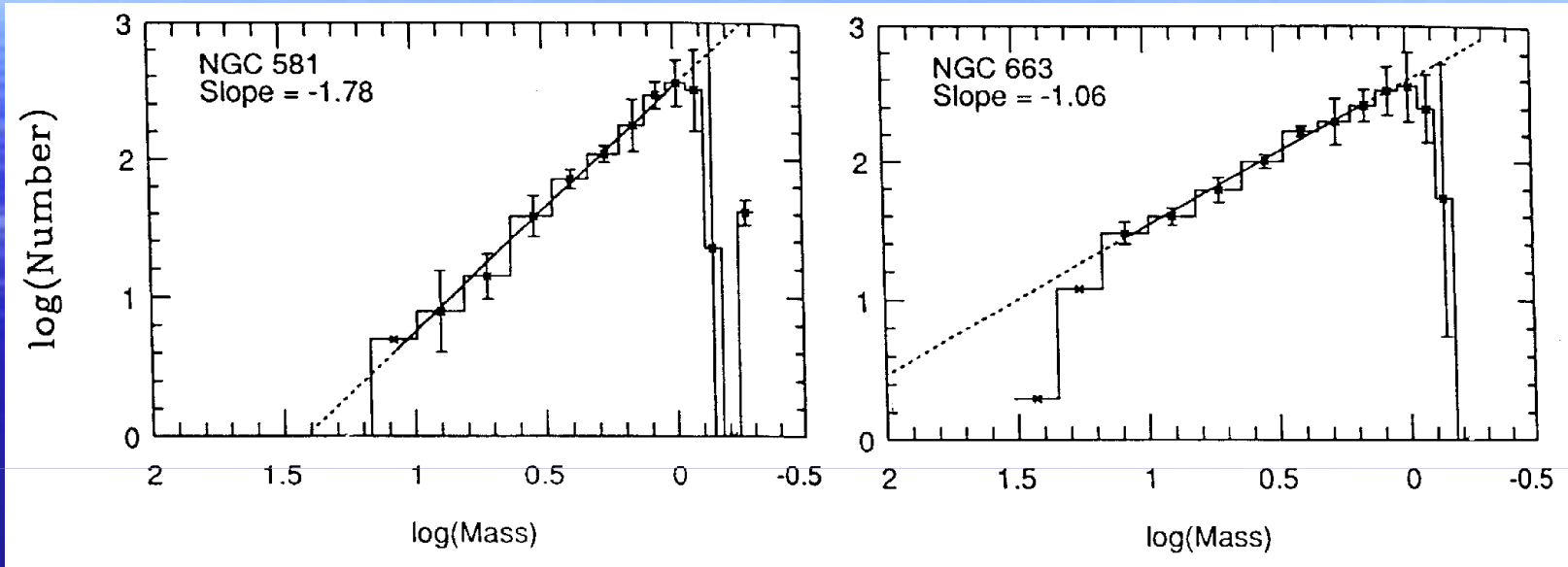


**Nubes interestelares,
formación estelar, y fractales.**

Néstor Sánchez
IAA-CSIC, aquimismo.

Motivación

Formación estelar: no entendemos casi ná
(ej. Qué determina la masa de las estrellas?)



Scalo (1998)

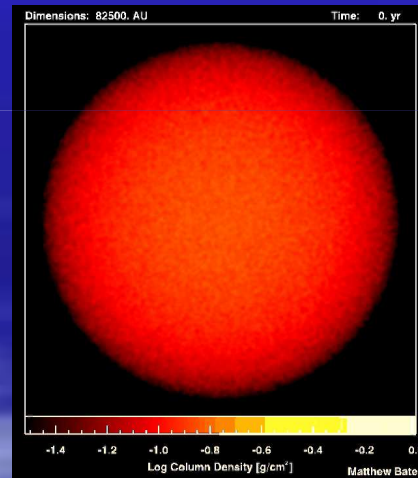
61 cúmulos abiertos y asociaciones
(relleno=VL, abiertos=LMC)

Motivación

Condiciones iniciales:
GMCs, clouds, clumps (estructura ISM)
Variables ambientales?

Formación estelar
→

Estrellas recién nacidas



Estructura jerárquica,
masas, tamaños, ...

???

IMF, distribución espacial, ...

Estudio/Caracterización Cuantitativa/Objetiva/Sistemática

Para (poder hacer) un análisis sistemático la caracterización debe ser cuantitativa.

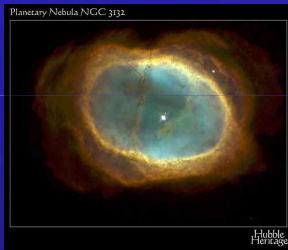
Eliminar la componente subjetiva y analizar muchos datos.



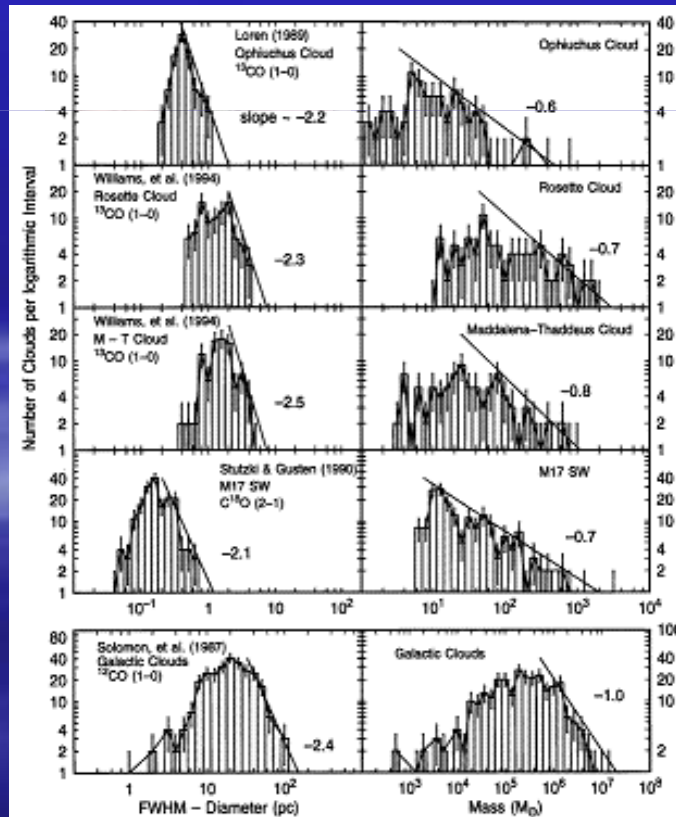
bonita?



uf, complicada!



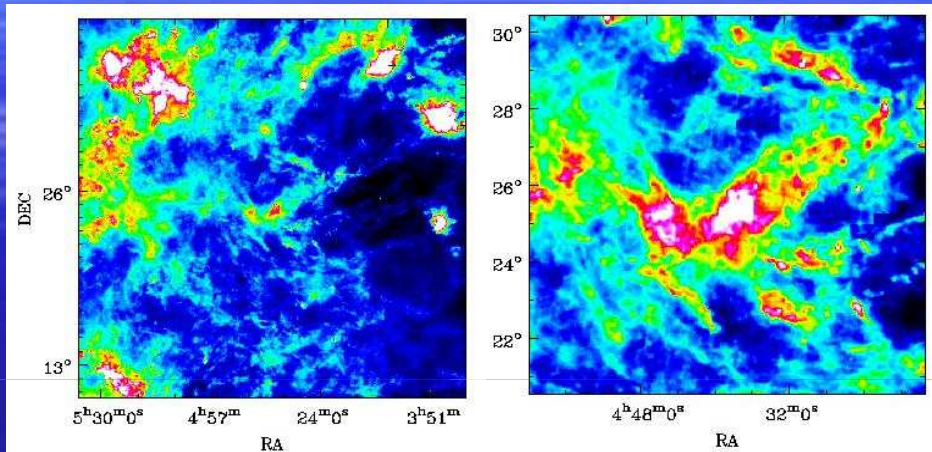
redondita?



Y la herramienta de análisis debe ser "robusta"...

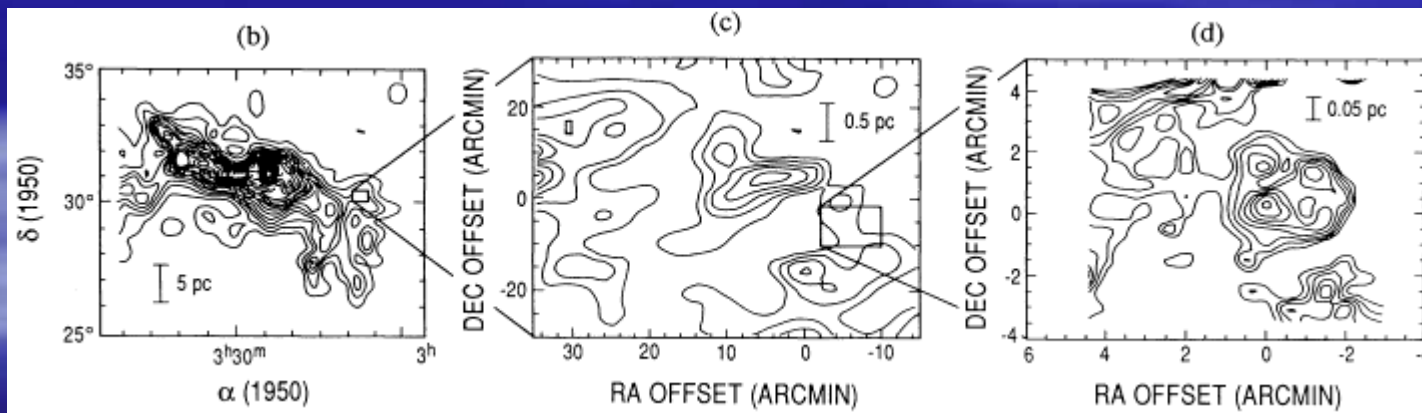
Elmegreen & Falgarone (1996).

- Mapas de nubes cercanas muestran una estructura jerárquica y auto-similar



Nube Molecular Taurus
(IRAS 100 μ emisión)

Complejo Taurus: emisión integrada
en ^{12}CO (Falgarone et al. 1991)



Jerarquía + auto-similaridad: fractales! → Análisis fractal

¿Qué es un fractal?

- The Fractal Geometry of Nature (Mandelbrot, 1983).

<http://www.math.yale.edu/mandelbrot/>

Definición matemática:

Objeto cuya dimensión de Hausdorff (dimensión fractal) excede su dimensión topológica (%@#!!!?).

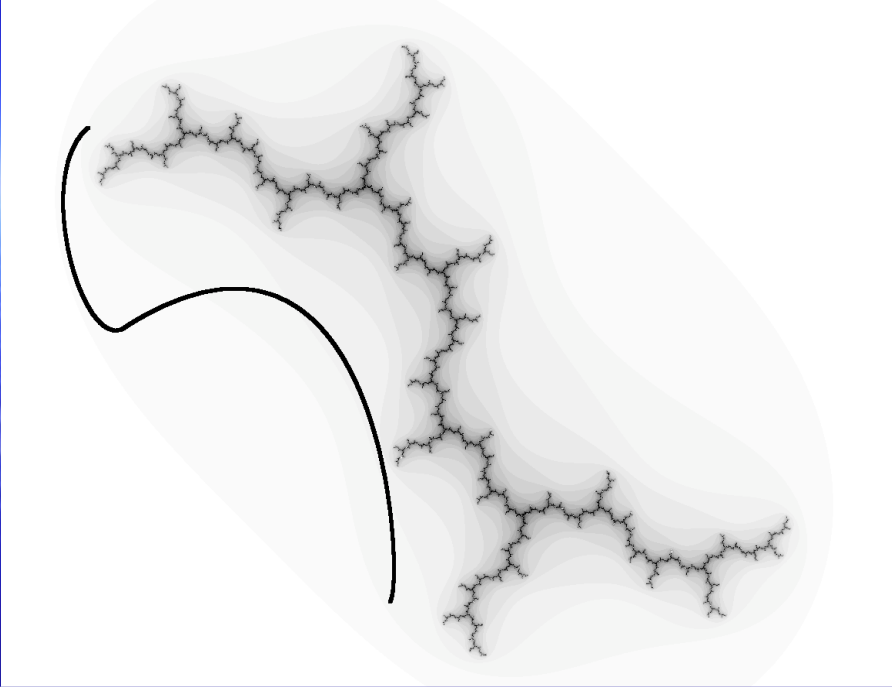
$$\dim_H(F) := \sup\{s : \mathcal{H}^s(F) = \infty\} := \inf\{s : \mathcal{H}^s(F) = 0\}$$

$$\mathcal{H}^s(F) = \lim_{\delta \rightarrow 0} \mathcal{H}_\delta^s(F)$$

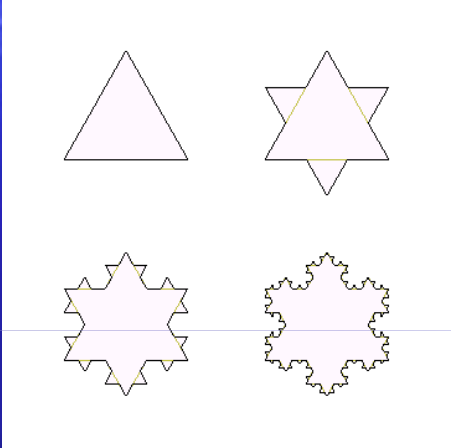
$$\mathcal{H}_\delta^s(F) = \inf\left\{\sum_{i=1}^{\infty} |U_i|^s\right\}$$

¿Qué es un fractal?

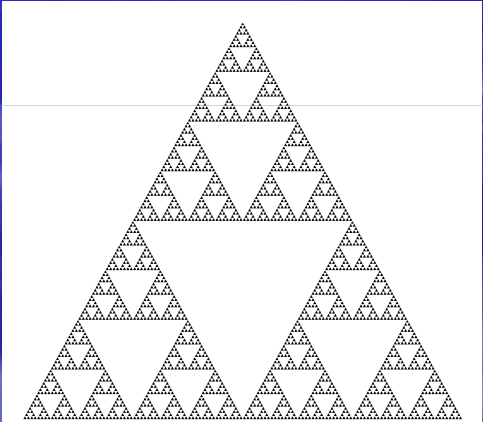
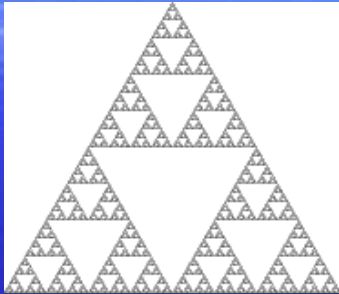
- The Fractal Geometry of Nature (Mandelbrot, 1983).
<http://www.math.yale.edu/mandelbrot/>
- Un fractal **usualmente**:
 - Es muy irregular (en términos euclidianos)
 - Tiene estructura (fina) a pequeñas escalas
 - Se construye a partir de una definición recursiva
 - Es **auto-similar** (al menos aproximadamente)



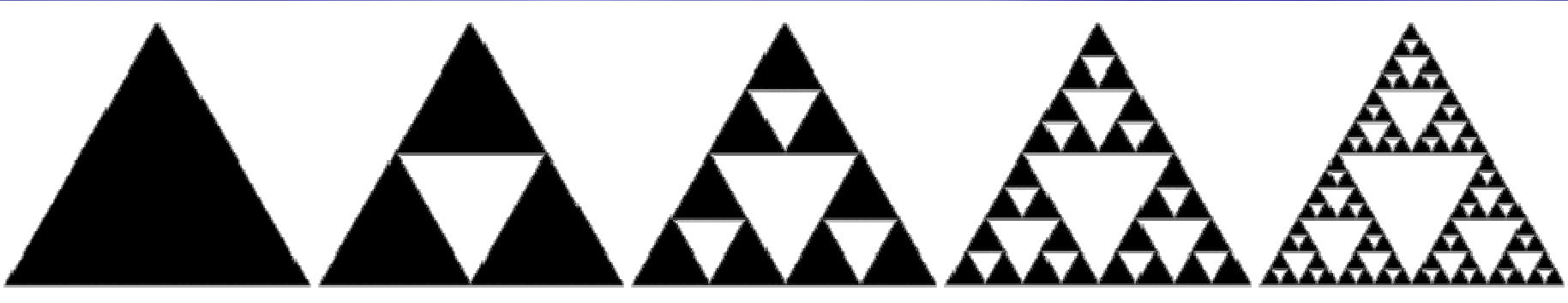
Dendrita de Julia



Curva de Koch



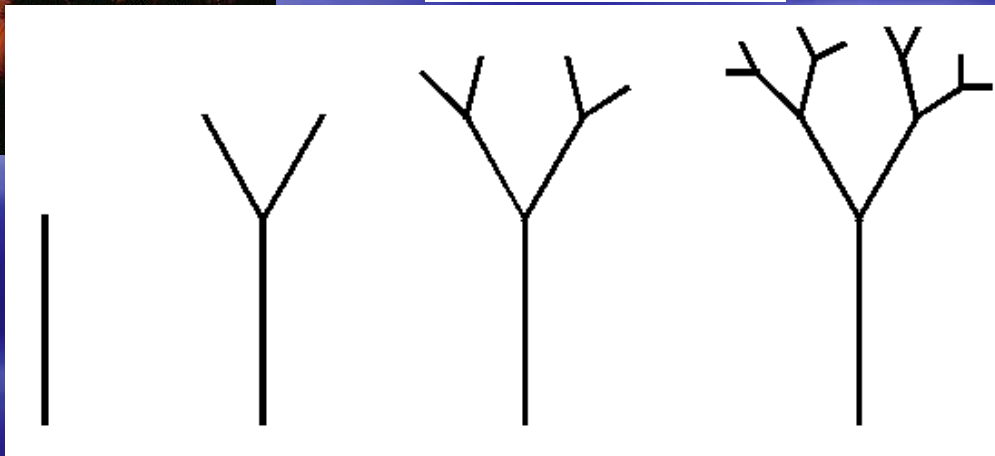
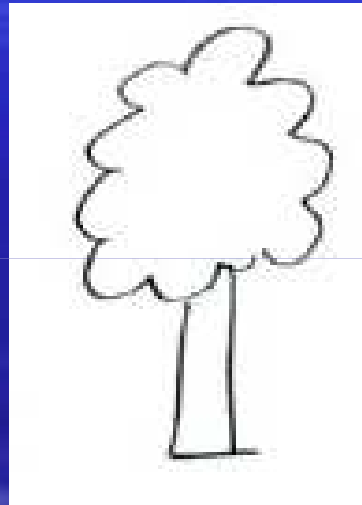
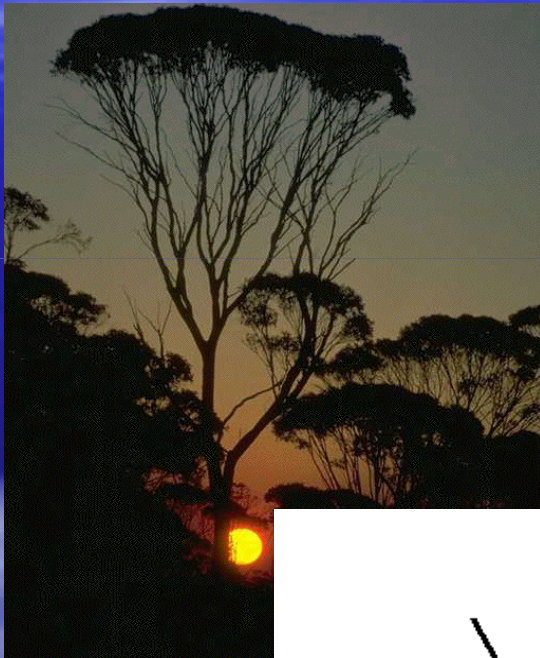
Triângulo de Sierpinski

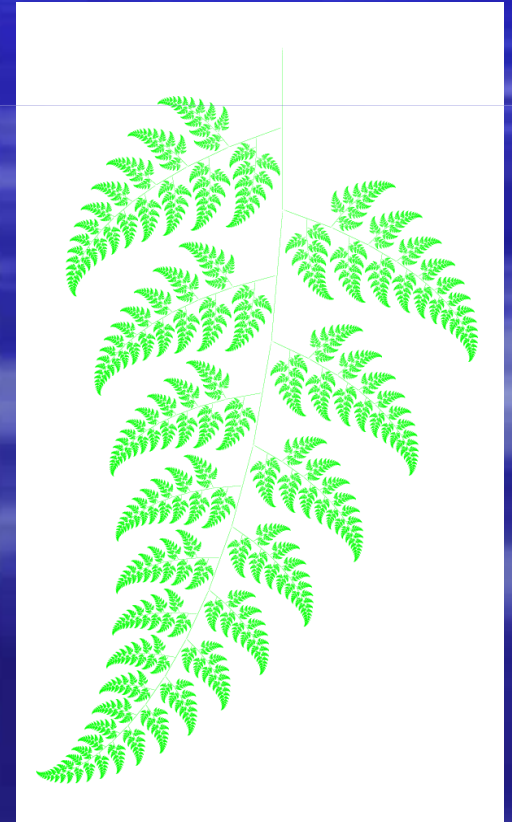
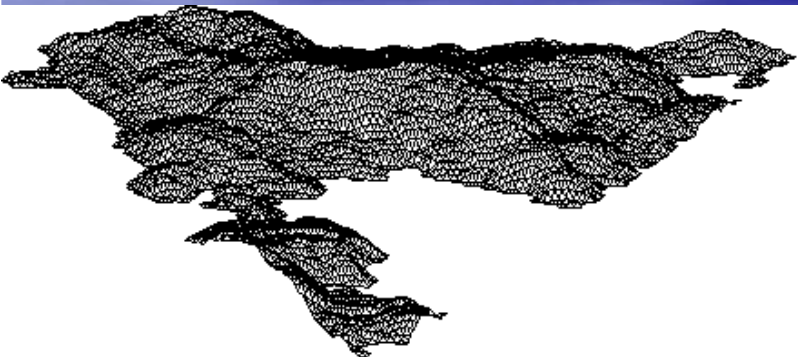
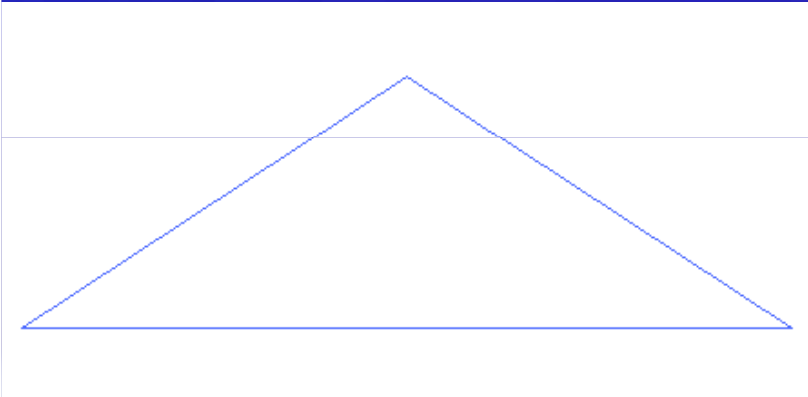
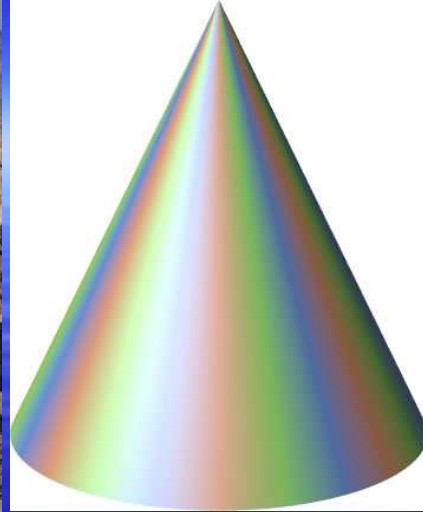


Los fractales pueden ser útiles a la hora de tratar de modelizar y/o entender la naturaleza.

La naturaleza esta llena de objetos tipo-fractal.

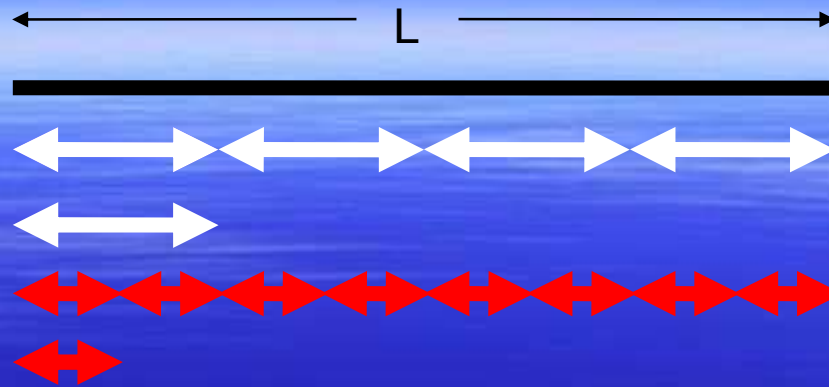
Las nubes no son esferas, las montañas no son conos, las líneas costeras no son círculos, las cortezas no son suaves, y tampoco los rayos viajan en línea recta (Mandelbrot, en la introducción de *The Fractal Geometry of Nature*).





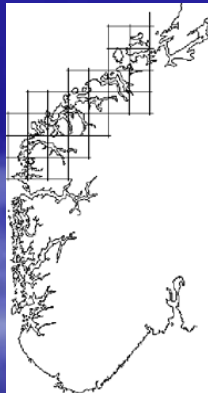
Dimensión fractal

$$N = L / R$$



En el ensayo de Mandelbrot hay un capítulo titulado:
 -"How long is the Coast of Britain"
 "How long is the Coast of Venezuela"

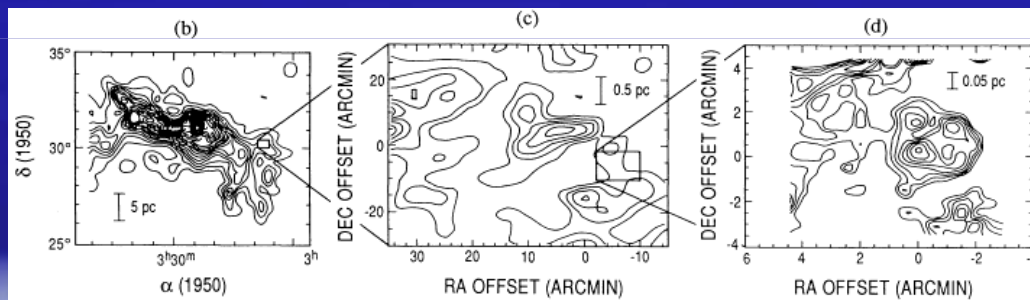
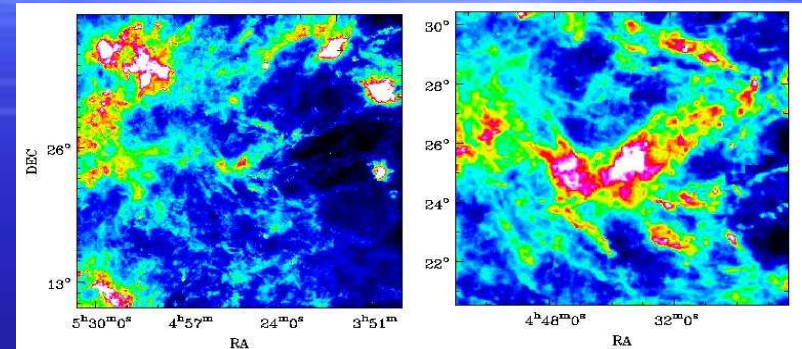
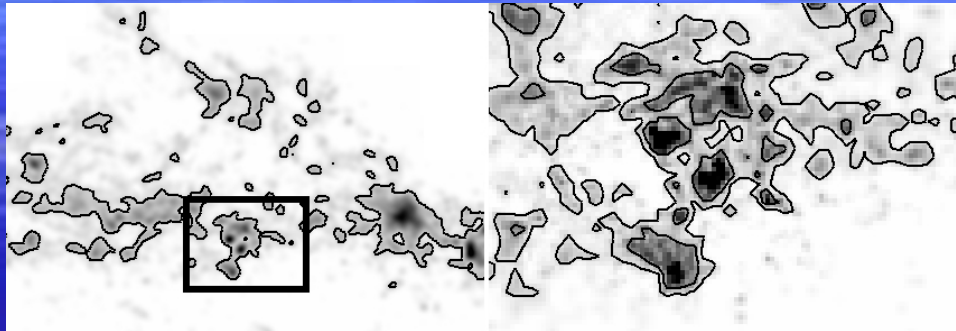
$$N = L / R^{Df} , Df > 1$$



- Costa de Noruega: $Df=1.52$
- Costa de Australia: $Df=1.13$
- Costa de Venezuela: $Df=???$



- Mapas de nubes cercanas muestran una estructura jerárquica y auto-similar

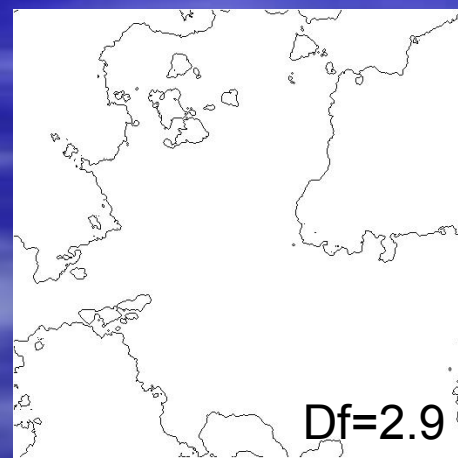
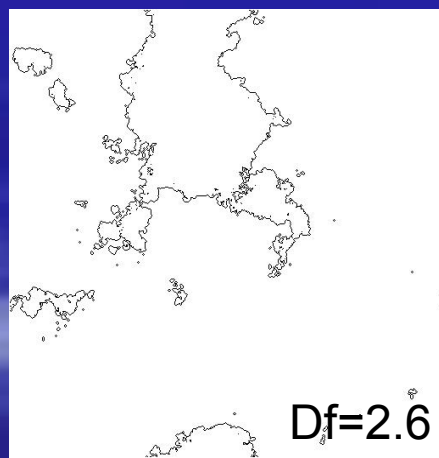
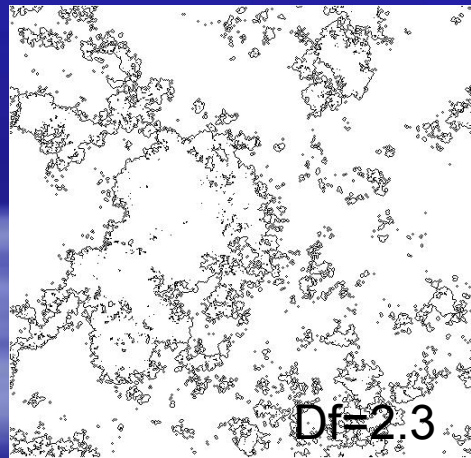
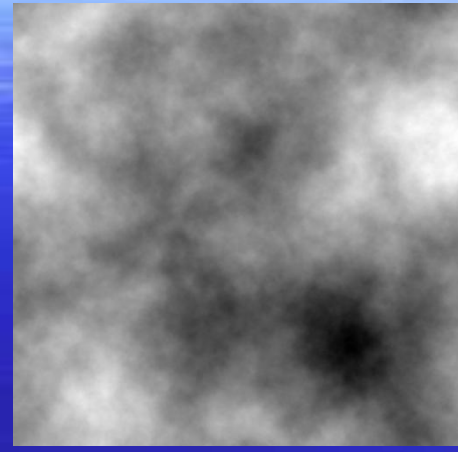
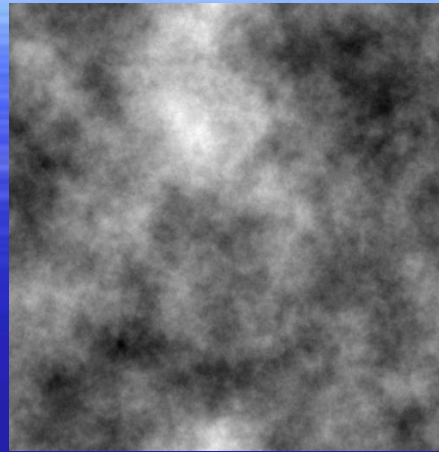
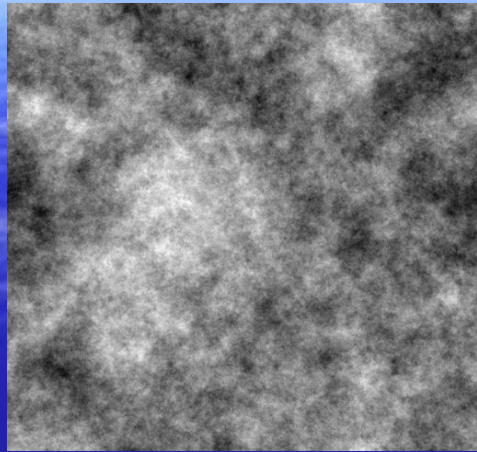


→ Estructura fractal subyacente

Podemos cuantificar la estructura (el grado de complejidad, de irregularidad, de “clumpiness”) a través de su **dimensión fractal** (D_f).

Nubes fractales simuladas:

13/24

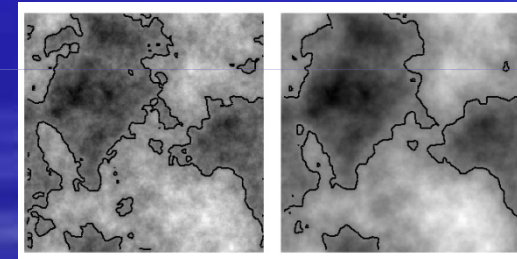
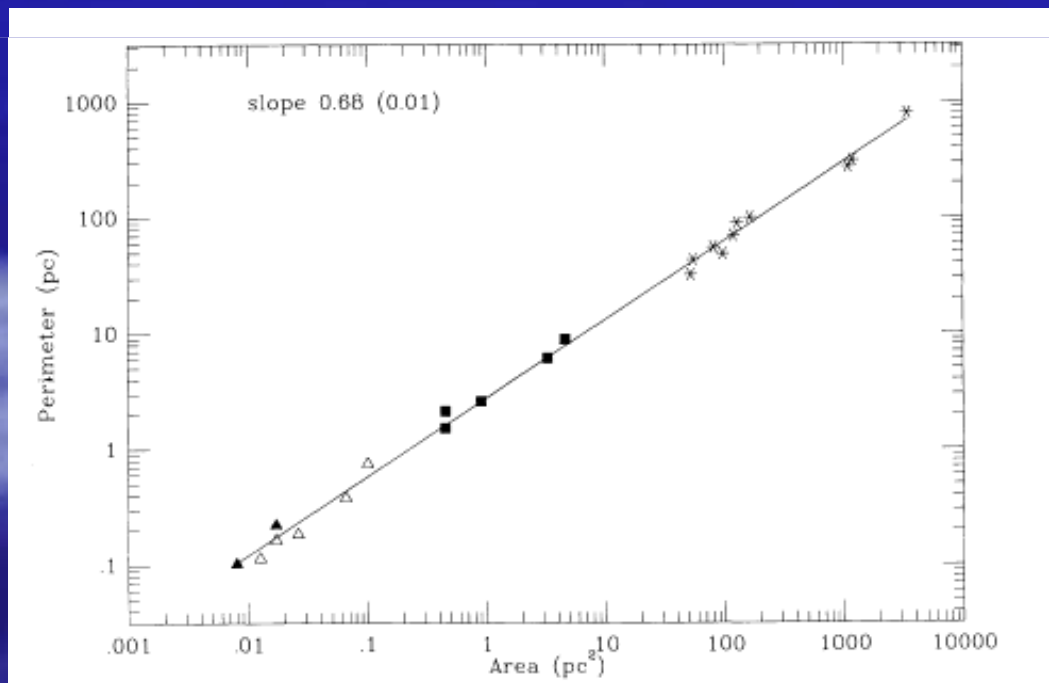


Estimadores de la dimensión fractal:

$$F \sim x^{D_f}$$

Area-Perímetro-area:	$P \sim A^{D_{per}/2}$
Masa-radio:	$M \sim r^{D_m}$
Integral de correlación:	$C \sim r^{D_c}$

La mayoría de los estudios usan D_{per} :



$$P \sim A^{D_{per}/2}$$

$$\Rightarrow D_{per} = 1.36$$

Complejo Taurus: emisión integrada en ¹²CO
(Falgarone et al. 1991)

Estado “actual”:

- Los mapas observados indican que:

$$D_{per} \cong 1.35$$

- Lo cual “sugiere” que en 3D:

$$D_f = D_{per} + 1 \text{ (e.g. Beech 1992).}$$

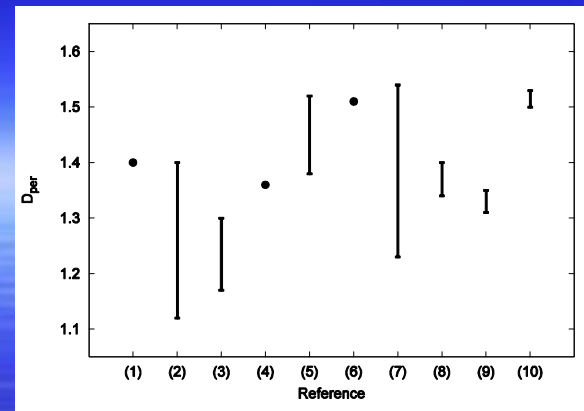
- Por tanto, $D_f \cong 2.35!!!$ (valor universal)

Difusión turbulenta en un fluido incompresible da $D_f \cong 2.3$ (Meneveau & Sreenivasan 1990).

TODO Ok!!!

Df (ISM) = ???

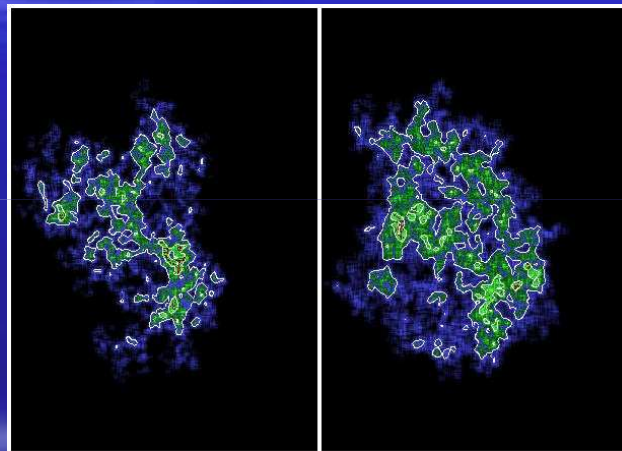
Df(ISM) \approx 2.3 = universal !!!???



Enfoque empírico: simulación (nubes) + proyección + calcular Df

miprogramita.f

Df bien definida

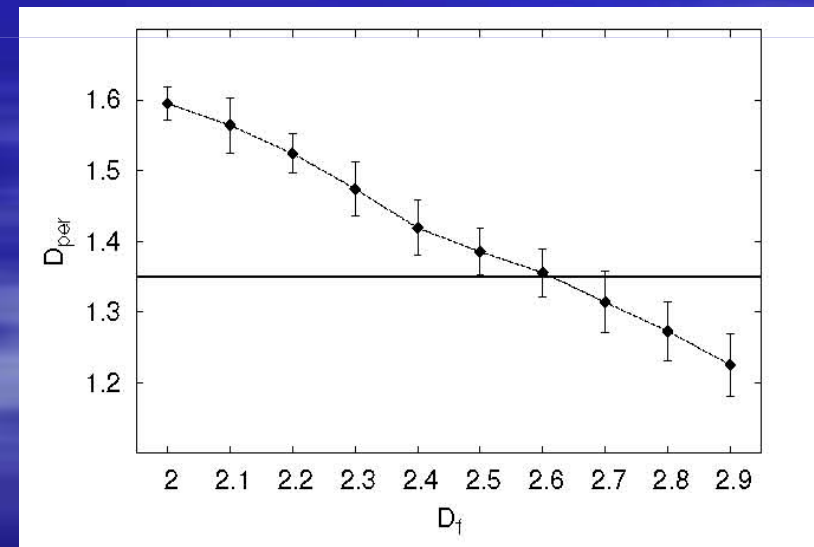


Df=2.3

Df=2.6

miotroprogramita.f

Df(imagen)
Dper, Dc, Dm



Dper=1.35 \rightarrow Df = 2.6 +/- 0.1 !!!

Estado “actual”:

- Los mapas observados indican que:

$$D_{per} \cong 1.35$$

- Lo cual “sugiere” que en 3D:

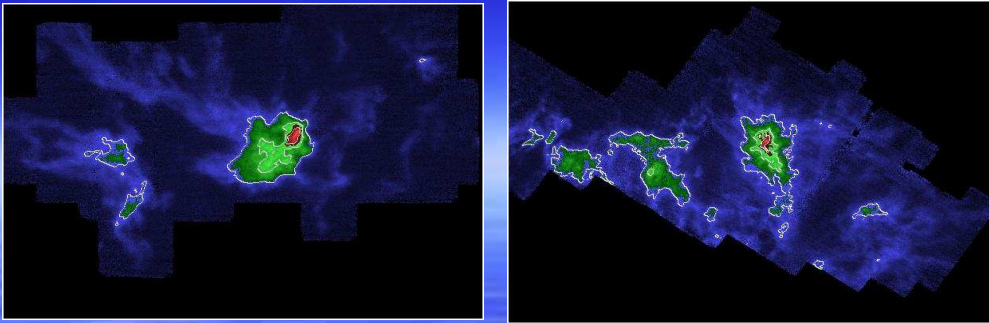
$$D_f = D_{per} + 1 \text{ (e.g. Beech 1992).}$$

- Por tanto, $D_f \cong 2.35!!!$ (valor universal)

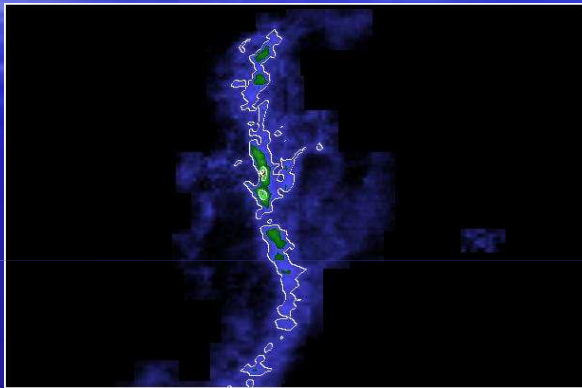
Difusión turbulenta en un fluido incompresible da $D_f \cong 2.3$ (Meneveau & Sreenivasan 1990).

TODO Ok???

Mapas en 13CO

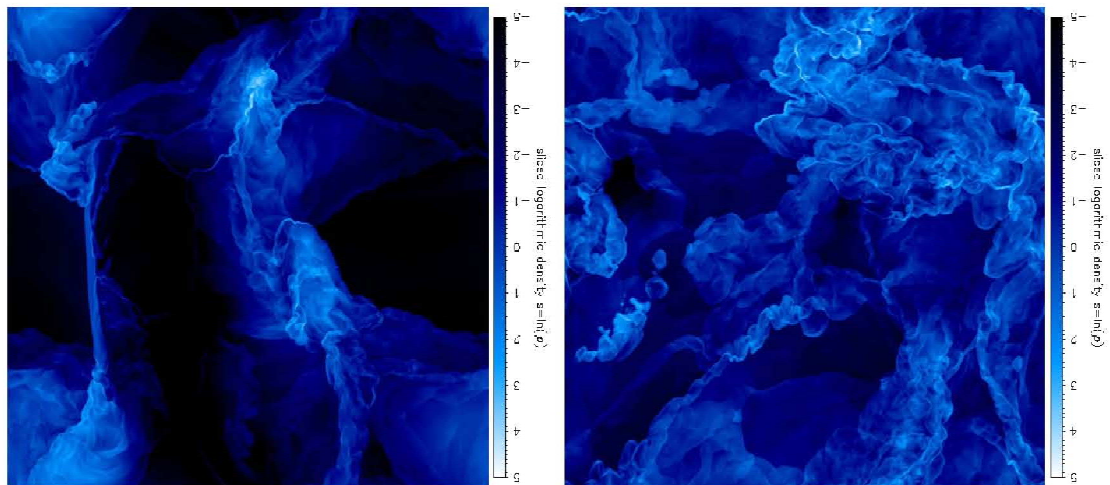
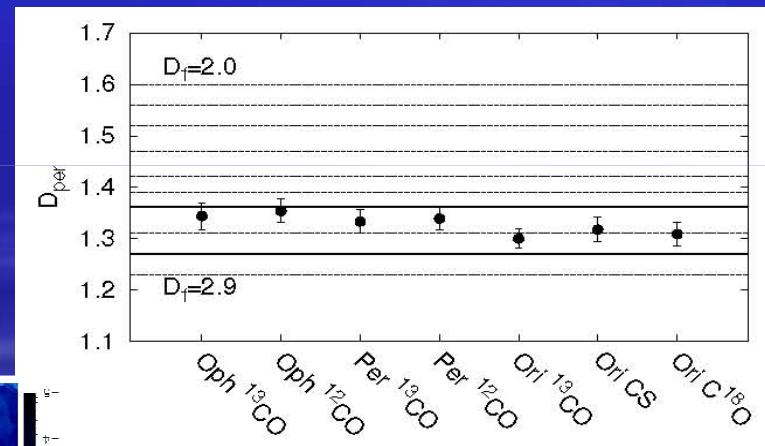


Ophiuchus, Perseus (COMPLETE, Ridge et al. 2006)



Orion (Nobeyama, Tatematsu et al. 1993)

$D_f=2.6-2.8$ (Universal?)



$D_f=2.3$ vs $D_f=2.6$

Federrath et al. 2009:

$D \sim 2.3$ forcing compressivo

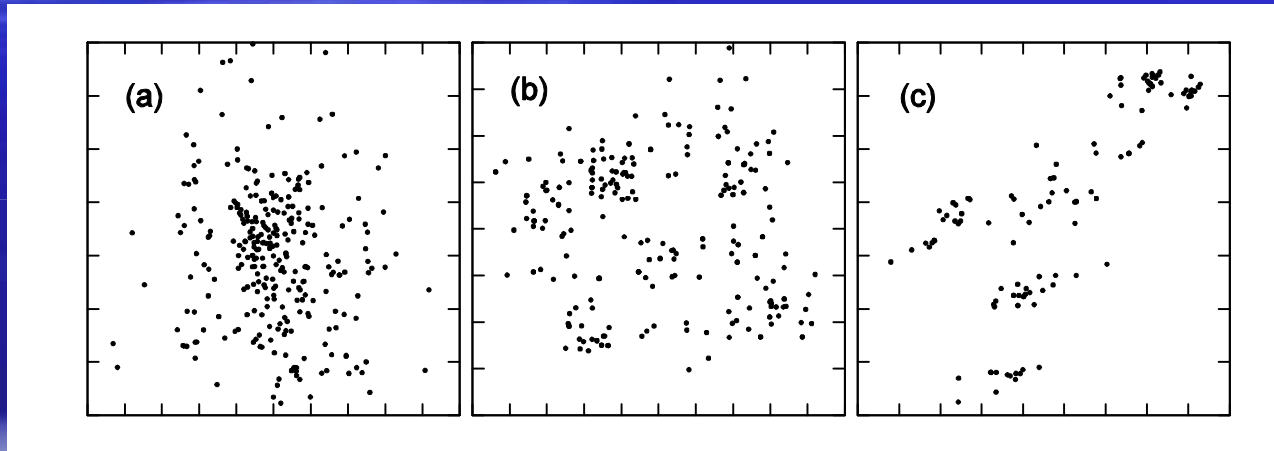
$D \sim 2.6$ forcing solenoidal ($M=5.5$)

Cúmulos estelares / asociaciones

Estado inicial de un cúmulo?

Estructura de un cúmulo: condiciones iniciales + evolución dinámica (temprana)

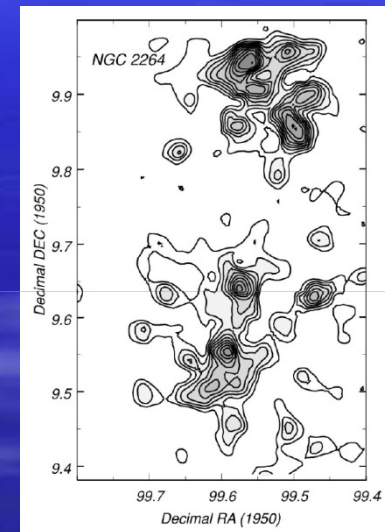
ISM fractal (jerárquico, clumpy)



IC 348

IC 2391

Taurus



Fuentes infrarojas en NGC 2264 (Lada+2003)

“Creencia” generalizada:

Estructura inicial clumpy

→ relajación dinámica (subestructura se disuelve)

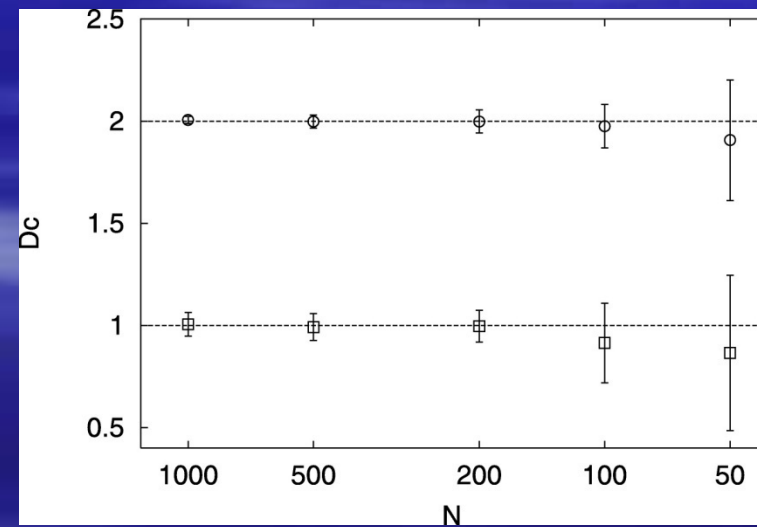
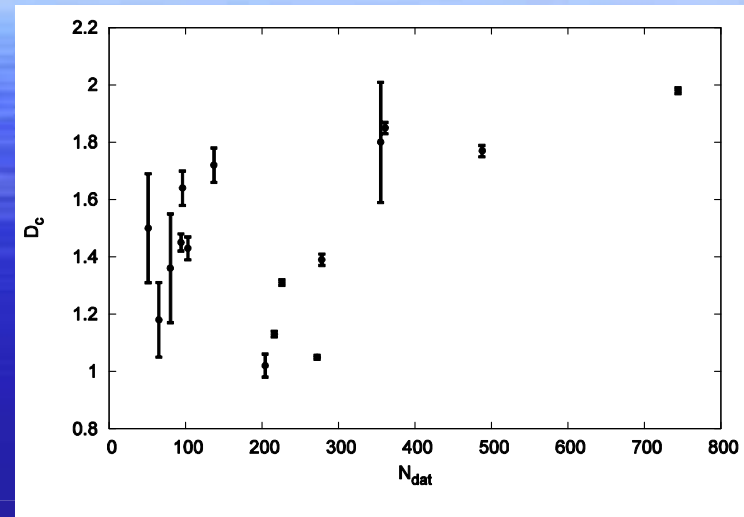
→ Perfil radial de densidad

Dimensión fractal en cúmulos jóvenes

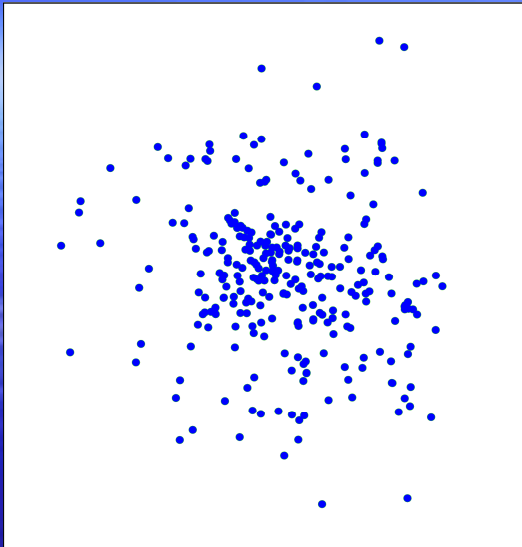
Table 2: Summary of correlation dimensions for the distribution of stars in clusters (literature results that use the MSDC technique).

Ref.	N_{dat}	D_c	Cluster
(1)	>121	1.38	Taurus-Auriga
(2)	80	1.36 ± 0.19	Taurus
	51	1.50 ± 0.19	Ophiuchus
	355	1.80 ± 0.21	Trapezium
(3)	361	1.85 ± 0.02	Orion OB
	488	1.77 ± 0.02	Orion A
	226	1.31 ± 0.01	Orion B
	96	1.64 ± 0.06	Ophiuchus
	103	1.43 ± 0.04	Chamaeleon I
	94	1.45 ± 0.03	Chamaeleon
	278	1.39 ± 0.02	Vela
	65	1.18 ± 0.13	Lupus
	(4)	744	1.98 ± 0.01
(5)	137	1.72 ± 0.06	Chamaeleon I
	216	1.13 ± 0.01	Taurus
(6)	204	1.02 ± 0.04	Taurus
(7)	272	1.049 ± 0.007	Taurus-Auriga

Reference index: (1) Larson (1995); (2) Simon (1997); (3) Nakajima et al. (1998); (4) Bate et al. (1998), their first data set; (5) Gladwin et al. (1999); (6) Hartmann (2002); (7) Kraus and Hillenbrand (2008).



Clumpy versus smooth structure



IC 348. Luhman+2003.
age(webda)~40 Myr

Radial clustering

Profile fitting:

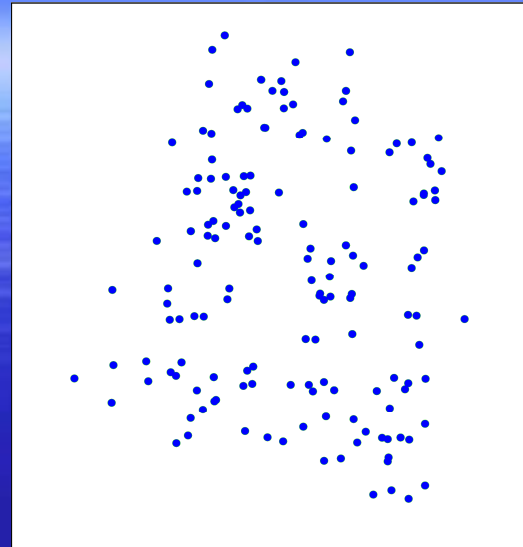
Karchenko:

$R_c=3.6$ arcmin

$R_t=7.8$ arcmin

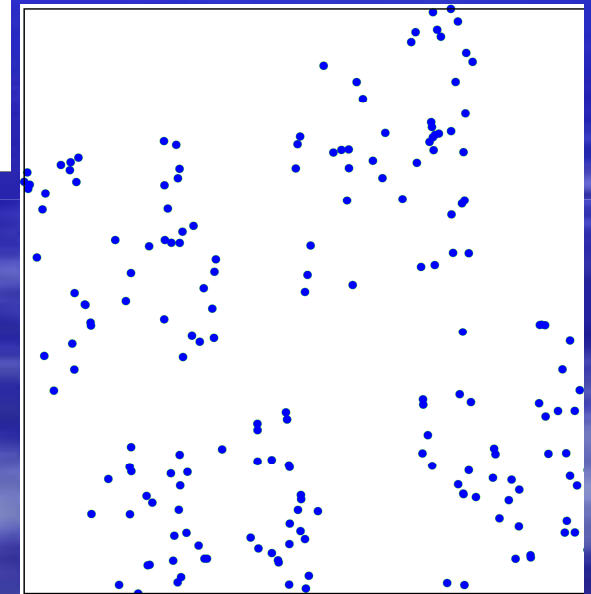
Webda:

$R_{cluster}=4$ arcmin



IC 2391. Barrado+201.
age(webda)~50 Myr

Fractal clustering

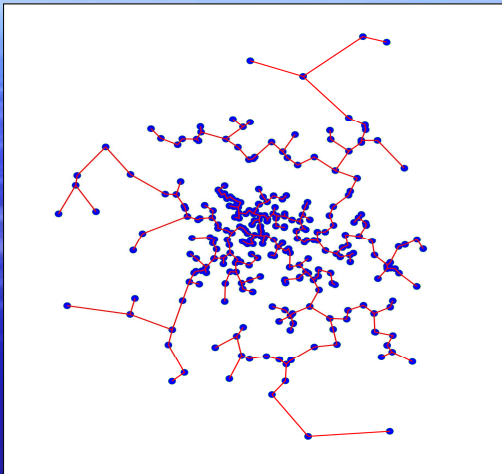


Simulated fractal
Fractal dimension...

First step: we need to distinguish (but not by eye!)
between fractal clustering and density gradient

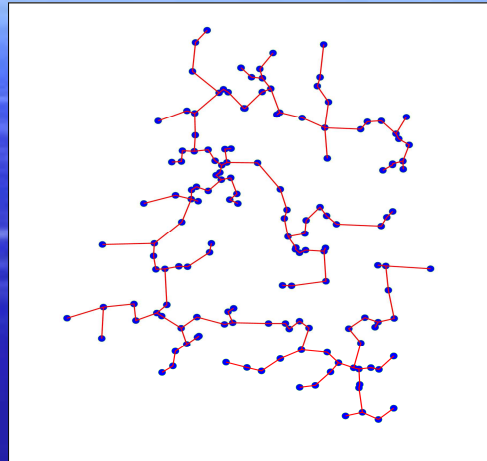
Minimum spanning tree: Structure parameter (Q)

IC 348. Luhman+2003.

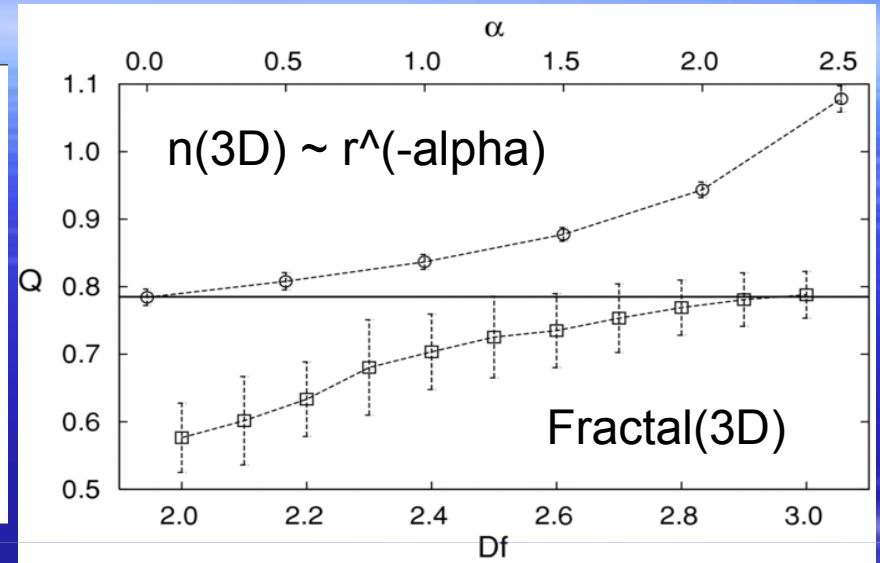


Q=0.98

IC 2391. Barrado+2001.



Q=0.66



Cartwright+2004:

$$Q = \frac{\bar{m}}{\bar{s}}$$

---> (normalized) mean length of the branches
 ---> (normalized) mean separation between stars

Homogeneous distribution of points: $Q \approx 0.8$

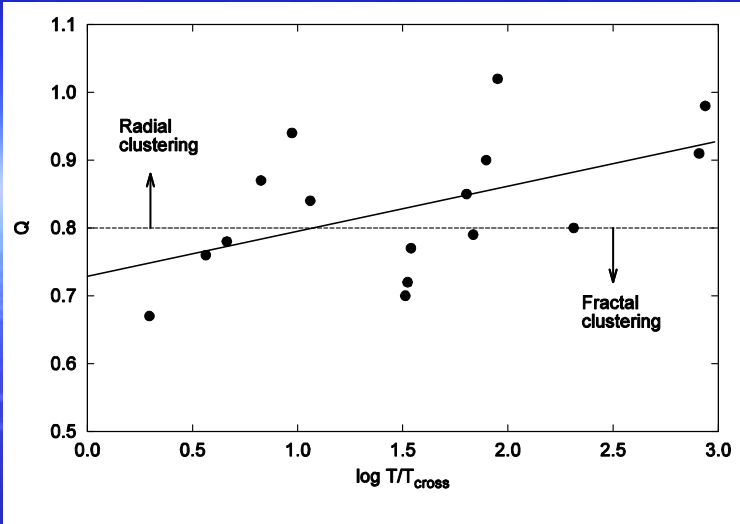
- Central clustering (radial) --> Q increases ($Q > 0.8$)
- Fractal clustering (clumpy) --> Q decreases ($Q < 0.8$)

MST (open clusters)

Table 1. Properties of the sample clusters.

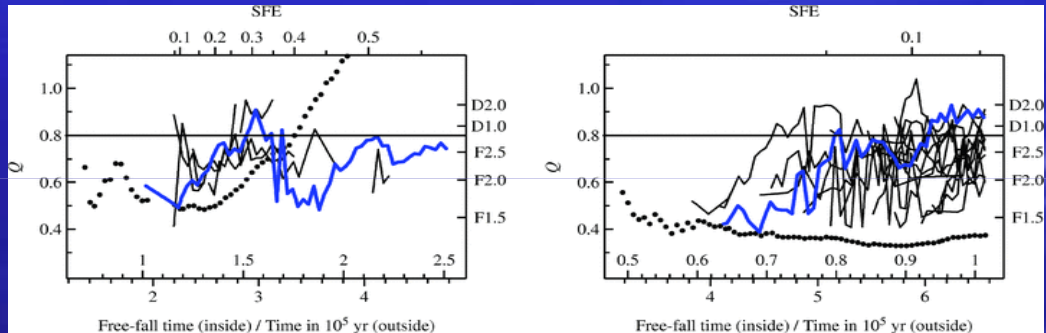
Name	log T	D	N _c	R _c	R _t	Q	D _c
IC 2391	7.661	175	62	1.46	2.65	0.77	1.74 ± 0.20
M 11	8.302	1877	289	1.98	4.49	1.02	...
M 34	8.249	499	181	0.11	1.73	0.80	2.04 ± 0.05
M 67	9.409	908	354	2.21	5.92	0.98	...
NGC 188	9.632	2047	1459	2.90	10.57	0.91	...
NGC 581	7.336	2194	526	1.38	11.86	0.76	1.79 ± 0.06
NGC 1513	8.110	1320	156	1.55	7.73	0.72	1.82 ± 0.09
NGC 1647	8.158	540	683	1.23	8.86	0.70	1.94 ± 0.02
NGC 1817	8.612	1972	277	3.39	11.97	0.79	1.94 ± 0.04
NGC 1960	7.468	1318	311	2.96	8.77	0.87	...
NGC 2194	8.515	3781	228	3.17	10.31	0.85	...
NGC 2548	8.557	769	168	2.61	9.16	0.90	...
NGC 4103	7.393	1632	799	0.72	10.74	0.78	1.85 ± 0.04
NGC 4755	7.216	1976	196	1.11	3.50	0.94	...
NGC 5281	7.146	1108	80	0.62	2.44	0.84	...
NGC 6530	6.867	1330	145	1.43	7.47	0.67	1.74 ± 0.09

T: cluster age (Myr), D: distance (pc), N_c: number of members, R_c: core radius (pc), R_t: tidal radius (pc), Q: structure parameter, D_c = correlation dimension.



- Spatial substructure in old clusters
- Q increases with $\sim T/T_{cross}$
- (as it should be for each cluster!)

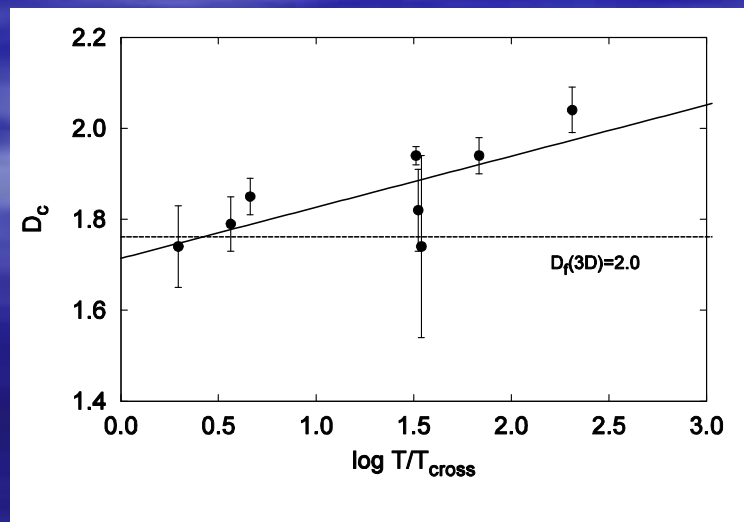
Maschberger+2010



Lowest Dc value = 1.74

→ Df(3D) ~ 1.9-2.2 << 2.6 ± 0.1 (ISM)

- Perhaps some clusters may develop some kind of substructure starting from an initially more homogeneous state (Goodwin+2004).
- Another possibility is that the Df in the Galaxy does not have a universal value and therefore some regions form stars distributed following more clustered patterns.



Resumiendo...

- Análisis fractal (guay):
estructura del ISM + distribución de estrellas
- MST – Q: cúmulos (clustering radial y/o fractal)
- $D_f(\text{ISM}) \approx 2.7 \pm 0.1$ (> 2.3) (universal?)
- $D_f(\text{estrellas}) = \dots$ (efectos evolutivos), $D_f(t=0) \ll 2.7???$

Futuro:

- Otras herramientas de análisis cuantitativo
(Sandra?, si viniste estas viendo esto!)
- Aplicación sistemática (ISM + estrellas)
- Otros sistemas/escalas espaciales

Gracias...

