Possible patterns in the distribution of planetary formation regions

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Accepted Received

ABSTRACT

Eris, an object larger than Pluto, is known to reside in the transneptunian region further away than Pluto. One can wonder whether its semimajor orbital axis fits in a generalized Titius-Bode law, in the same way as Pluto does. We performed a new least squares fit to a generalized Titius-Bode law including Eris and found that not only does Eris fit in the trend, but also that the correlation coefficient improves. In addition, there is a remarkable symmetry of the location of the planetary formation regions with respect to Jupiter when the natural logarithm of the heliocentric distance is used as the metric. The issue of whether the observed patterns have some physical meaning or are due to mere chance is addressed using a Monte Carlo approach identical to that by Lynch (Lynch, P. On the significance of Titius-Bode law for the distribution of planets. Mon. Not. R. Astron. Soc. 341, 1174-1178. 2003). Although the probability of chance occurrence is highly dependent on the way in which the random configurations of synthetic planetary systems are selected, we find that in all reasonable scenarios of random planetary systems the probability of chance occurrence of the observed patterns is small (below 1% in most cases). If the trend were used as a prediction tool, one might expect another planet or dwarf planet or a swarm of bodies with semimajor orbital axis of 120 AU ± 20 AU. Simple calculations show that the protoplanetary nebula most likely had enough mass to allow the accretion of at least a dwarf planet at that distance. We also found that if the surface density of the nebula decayed with heliocentric distance (r) as a power of -2, the regular spacing in ln(r) in the solar system could be a natural consequence of the existence of a threshold mass for planetary formation.

Key words: Kuiper Belt – Solar System: general – Solar System: formation.

1 INTRODUCTION

Although not all possible random planetary configurations are physically stable, because of close encounters, etc., one might expect that the average distances of the planets to the Sun could be more or less random, as there are no a priori reasons why planets should have formed preferentially at some locations or have evolved to specific locations. However, there may be indications that the planet formation regions were not random to start with. One of those indications might be what we call “generalized Titius-Bode laws”.

As is widely known, a possible trend in the planetary distances was proposed very long time ago. This was the famous Titius-Bode law, which is currently stated in the form of the equation: $a = 0.4 + 0.3 \times 2^n$, where $a$ is semimajor orbital axis in AU and $n$ is -∞ for Mercury, 0 for Venus, 1 for the Earth and so forth. This old “law” was used as a prediction tool to discover objects like Ceres and Uranus. The Titius-Bode law in the above given form was not only rather arbitrary in the choice of the value of $n$ for Mercury, but as is well known, it also failed to reproduce the positions of Neptune and Pluto. Therefore, it is usually discarded as a law with some physical meaning, and is usually thought of as a mere curiosity that enters the field of “numerology” rather than science.

Nevertheless the semimajor axes of the orbits of Mercury, Venus, Earth, Mars, Ceres, Jupiter, Saturn, Uranus, Neptune and Pluto approximately follow a geometric progression, or a regular spacing in ln(r) (where r is the average radial distance to the sun), or an exponential law. We refer to all these equivalent expressions as to “generalized Titius-Bode laws”. Using the progression terminology, one
can state that the planets follow a geometric progression in which the ratio is not exactly 2, but 1.711, as noted by e.g. Richardson (1945) quite some time ago. In other words, the distances of the above mentioned objects do not follow the Titius-Bode law, but appear to approximately follow a general trend given by the progression \( a = 0.21 \times 1.71^n \) or equivalently, by the equation \( a = \exp(-1.51 + 0.52n) \), where \( a \) is semimajor axis in AU and \( n \) is the number in the sequence. The correlation coefficient of the fit is larger than 0.996, which is remarkably high and the chi-square value is 0.1568. The significance is also very high, larger than 0.9999.

This sort of generalized Titius-Bode laws or trends in planetary distances could be a mere coincidence, but the chances of such an arrangement happening by pure serendipity are difficult to estimate and are small or large depending on the method used to define a random set of planetary systems. Therefore the possibility of a physical explanation for the observed distribution remains open (Lynch, 2003).

Some researchers have claimed that even if there were statistical significance, there is no physics behind this progression other than scale invariance and rotational symmetry (Graner and Dubrulle, 1994), whereas others have proposed many different physical scenarios to explain the trend (see e.g. Nieto (1972) for a review of theories until 1970). On the other hand, Hayes and Tremaine (1998) claimed that the significance of generalized Titius-Bode laws is that stable systems tend to be regularly spaced, and conjectured that their conclusion could be strengthened by making gigayear orbit integrations to reject unstable planetary configurations.

Eris (formerly known as 2003UB313) is a Transneptunian Object whose size was determined to be larger than that of Pluto (Bertoldi et al., 2006) soon after its discovery and whose mass is most likely larger than that of Pluto as well. Besides, Eris is much further away than Pluto and belongs to a different population. Eris is the largest member of the so-called Scattered Disc. Also, some surface properties of Eris appear to be similar to those of Pluto and thus, this object should count in the analysis of planetary distances as well. Therefore, one can wonder how the semimajor axis of Eris orbit would fit in the exponential trend. In this paper we add the object Eris to the analysis and study the emerging patterns from different points of view.

2 THE DISTANCES OF THE PLANETARY FORMATION REGIONS

We will use the term “planetary formation region” or “planetary belt” to mean an area where there was a belt of objects until a planet formed and therefore (by the 2006 IAU definition of planet) cleared the neighbourhood of objects, or an area where planetary formation was not completed for whatever reason and therefore a belt of bodies still remains. Those areas approximately correspond to the locations where the following objects reside: Mercury, Venus, Earth, Mars, Ceres, Jupiter, Saturn, Uranus, Neptune, Pluto and Eris.

In addition, we will use the word “planets” (within quotes) to mean the largest body in each planetary belt. Even though this is not the official definition of planet, according to the 2006 IAU Praga’s resolutions, it would be cumbersome to refer to the 11 objects mentioned above by using the expression “planets and dwarf planets” and therefore we prefer to use one simple word like “planet”, although whenever this word is not used in the IAU sense, it will be given within quotes.

If one plots the semimajor axis of the orbits of the largest body in each planetary formation region, arranged in successive order, one can notice some sort of exponential behavior. Fig. 1 shows such a plot and table 1 lists the semimajor axes. To visually check whether an exponential behavior is really present one can take the natural logarithm of the semimajor axes and plot them versus “planet” number, which we do in Fig. 2. The overall exponential behaviour is readily seen because the overall trend is nearly a straight line in the ln plot. This also means that the planetary formation regions are almost equally spaced in ln(r). A linear fit is also shown in the figure. The best fitting least squares straight line gives a slope of almost 1/2 and an intercept of nearly -3/2, so that \( \exp((n - 3)/2) \) is close to the overall trend, although not exactly. The best fit is achieved with

\[
\exp(0.53074n - 1.51937)
\]  

The correlation coefficient of this fit is larger than 0.997, and the chi-square value is 0.1725. The probability that a value of chi-square as good as this should occur by chance is only 2.9 \times 10^{-7}, nearly an order of magnitude smaller than in the case without Eris. The same fit, but to a progression of the form \( \alpha = C + D^n \) results in \( C=0.22 \) and \( D=1.70 \pm 0.02 \). In other words, the separations of the “planets” follow a geometric progression of 1.7 instead of 2 as in the Titius-Bode law. Quite some time ago, an almost identical expression was already derived using all the “planets” except the then unknown Eris (e.g. Richardson, 1945). The correlation coefficient in that case was slightly worse than when Eris is added. Therefore we have shown here that Eris perfectly fits in the exponential trend and it improves the correlation coefficient slightly with respect to the case without Eris. The significance also improves. We can try to assess whether this is a coincidence and whether such a planetary configuration can arise from mere chance.

In order to estimate the chances of the planetary ar-

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\begin{align*}
\text{Figure 1. The semimajor axes of the orbits of the “planets” (in AU) are plotted versus “planet” number (diamonds). An exponential behaviour is suggested by the data. The exponential trend is more clearly seen when one takes the natural logarithm of the semimajor axis. This is done in Fig. 2.}
\end{align*}
\]
rangement being due to chance or not, we first followed the method of Murray and Dermott (1999) as outlined in Lynch (2003). From Monte Carlo simulations with $10^5$ samples, the chances that eleven distances be arranged in a similar way as in our solar system (in terms of residuals to an exponential fit) are not small (83%). The inclusion of Eris made a change with respect to the case in which Eris is not included. In that case the likelihood of a random configuration giving smaller residuals than the actual solar system is 87%.

Lynch (2003) also emphasized that the problem of assessing whether a given event is the result of chance or is so unlikely as to suggest a causative origin, is fraught with difficulty. From his analysis of the positions of the “planets” (not including Eris) he concluded that the estimated probability is very sensitive to the method of defining a random set of planetary systems and used several examples of random planetary systems with different radius exclusion recipes. We found that when we include Eris, all the same scenario that Lynch (2003) tested give smaller chances than when Eris is not used. There are many radius exclusion criteria that can be used to construct random planetary systems. Hayes and Tremaine (1998) used other more physically sound criteria to reject some of the log-random solar systems that they built. They concluded that the significance of generalized Titius-Bode laws appears to be that stable systems tend to be regularly spaced. Nevertheless, their log-random tests with the configuration in which Ceres is included resulted in better fits than the actual solar system in only 2-10% of the cases. The addition of another object like Eris would likely decrease the above figure even further and therefore the current solar system configuration would be far from random. However, they emphasized that the radius exclusion laws are somewhat arbitrary and they proposed that the best way to address the issue is by generating random systems and carrying out ~gigayear orbit integrations to reject unstable planetary configurations. The computing power nowadays allows us to address such a numerical approach, but we are still in the process of carrying out the orbital integrations. Very tentatively we can say that our simulations tend to indicate that the stable systems do not show a pattern of the type we see in our solar system. In order to test whether the dynamical evolution of a certain arrangement of masses placed at certain distances to the Sun results in long-term configuration resembling generalized Bode laws, we proceeded in the following way: We set 11 random masses (in the mass range of our solar system) at random distances from the Sun (constrained to be in the 0-75 AU range), and we studied the long-term evolution (100 MY to 1GY) of ~ 7000 random systems. All integrations were performed using Chambers (1999) hybrid symplectic/Bulirsch-Stoer integrator. Our very preliminary results show that stable configurations amount to about 1% of the total, and none of these displays a generalized Titius-Bode-like behaviour. Nonetheless, a more extensive analysis should be done in order to be able to make firm conclusions.

It is important to draw the attention to another interesting feature of the distribution of “planet” distances. Although the general pattern in the distances of the “planets” is remarkably reproduced by the exponential fit, there are still some residuals. The plot in Fig. 2 shows that there appears to be a symmetry of the data around Jupiter and the oscillation can be accurately fitted by means of a 5-degree polynomial fit, which is also shown in Fig. 2. The rms of the residuals of the polynomial fit is just 3%, which is quite good. We should stress that we are not interested in finding an equation that gives a nearly perfect fit to the “planet” distances, because this could be pure numerology and there are already complicated expressions that can lead to very small residuals. Our main goal by using the polynomial fit is to plot a smooth curve joining each data point, rather than using pure straight lines. This is done to guide the eye to easily see that there appears to be a symmetry around Jupiter. We plot the distances in ln(a) to Jupiter in Fig. 3. The symmetry is clearly seen, and although it is not perfect, it is quite remarkable. Again, we test the likelihood of this arrangement, by using Monte Carlo simulations with a $10^5$ random sample. The model is identical to that of Lynch (2003) but we add the constraint that the configurations be as symmetrical as that of our solar system. It turns out that for all the reasonable random planetary systems scenarios, the probability of the configuration being random was extremely small and even in the extreme $k=2/3$ scenario the probability was around 1%. Hence we are led to conclude that the planetary configuration we see is likely not random and there appears to be some physics behind it. The residuals are plotted in Fig. 4.

### Table 1. Planetary distances used in this work

<table>
<thead>
<tr>
<th>“Planet”</th>
<th>Mercury</th>
<th>Venus</th>
<th>Earth</th>
<th>Mars</th>
<th>Ceres</th>
<th>Jupiter</th>
<th>Saturn</th>
<th>Uranus</th>
<th>Neptune</th>
<th>Pluto</th>
<th>Eris</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (AU)</td>
<td>0.387</td>
<td>0.723</td>
<td>1.000</td>
<td>1.523</td>
<td>2.739</td>
<td>5.203</td>
<td>9.539</td>
<td>19.182</td>
<td>30.057</td>
<td>39.807</td>
<td>67.731</td>
</tr>
</tbody>
</table>

3 DISCUSSION

If the pattern shown here can be extrapolated, one might expect perhaps another planet or dwarf planet or even a swarm of bodies at around 126 AU (in semimajor axis) plus an oscillatory term, which is in the order of ±20 AU amplitude. Current TNO surveys cannot find objects this far, as shown by e.g. Jewitt et al. (1998), Allen et al. (2001), Trujillo et al. (2001). However, occultations of stars caused by small objects this far have been recently detected by Roques et al. (2006) in the visible. Chang et al. (2006) also detected occultations by TNOs in the X-ray domain, but unfortunately they could not assess the distance at which the events took place. Most likely the objects causing the X-ray occultations were also at the large distances of the Roques et al. (2006) results.

A Pluto-like object at an average distance of 120 AU would be approximately $(r_1/r_2)^4$ times fainter than Pluto, where $r_1$ and $r_2$ are the average distances to the Sun of the candidate object and Pluto respectively. This is because the brightness of an object decreases as the fourth power of its distance to the sun. Therefore, a Pluto-like object at an average 120 AU distance from the sun would be $(120/40)^4$
The natural logarithm of the semimajor axis of the "planets" (in AU) is plotted versus "planet" number (diamonds). The straight line is a least squares linear fit to the data and the other line is a 5-degree polynomial fit to the data. The overall pattern of the data is thus an exponential law, but there is a clear oscillation superimposed. Apart from the oscillation, a remarkable symmetry around Jupiter is readily seen, but it is more emphasized in Fig. 3.

Figure 3. Diamonds: absolute value of the natural logarithm of the semimajor axis of the "planets" minus the logarithm of the semimajor axis of Jupiter (in AU). A solid line is used to join the points. A remarkable symmetry around Jupiter, which is object number six, is readily seen. The dotted-dashed lines are just used to indicate the pairs of symmetric points around Jupiter.

One can try to address the issue of whether there was...
sufficient mass in the protoplanetary disc to accrete and form a planetary sized body at such distances.

The mass of a disc of radius $r$ within the protoplanetary disc can be calculated as

$$M(r) = \int_0^{2\pi} \int_0^r \sigma(r)rdrd\phi = 2\pi \int_0^r \sigma(r)rdr,$$

where $\sigma$ is the surface density $r$ is radius from the center (the sun) and $\phi$ is the azimuth angle variable.

There is no consensus as to what kind of dependence with $r$ the surface density had. Some arguments indicate power laws of indices -3/2 to -2 (e.g. Pollack et al., 1996). If one uses the surface density in Pollack et al. (1996) one finds that there was sufficient mass for another “planet”.

If we manipulate the integral so that we have $dlr$ rather than $dr$ we get:

$$M(\ln r) = 2\pi \int \sigma(r)r^2dlr,$$

and if we use $\sigma = \alpha r^{-2}$, where $\alpha$ is a constant, we get:

$$M(\ln r) = 2\pi \alpha \int dlr = 2\pi \alpha \ln r,$$

which means that we would get constant mass within constant intervals in ln r. In other words, if the surface density follows an $r^{-2}$ dependence, planetary formation trends like we show here would be the natural outcome of the existence of a threshold mass for planet formation. A similar interpretation is that every time the solar nebula has an annulus with sufficient mass, a “planet” grows naturally and the spacing of the “planets” is constant in ln r (equivalently, the distances of the “planets” follow an exponential trend). We can try to determine what that threshold mass would be by using several approaches. It turns out that the threshold mass is $\sim 55$ Earth masses, if one uses the total mass of the current planets to derive a lower bound on $\alpha$. Indeed, this would be a lower limit because the mass of the lost volatiles has not been added to the current planet masses. It must be emphasized that this is just a crude order of magnitude estimate. The exact $r$ dependence of the surface density of the protoplanetary nebula is not well known and therefore the analysis made here is only a speculation.

The question of whether other solar systems might show a similar pattern in the distance of planetary formation regions will only be resolved when large numbers of planets be found around many stars. Then, the possible role of massive Jupiter-like planets in shaping up the distribution of planets might be elucidated.

As a final side note, one can also point out that the trend in planetary formation distances could be used as a feature for a “planet candidate” to become “planet” or not. The requirement that an object fits in a progression is easy and yet restrictive enough so that the number of “planets” in our solar system would not be very large. A new definition of the term “planet” might be coined, as those objects whose orbital semimajor axis meet the exponential pattern shown here and also meet the requirements a) and b) of the IAU Praga 2006’s definition.

ACKNOWLEDGMENTS

This work was supported by contracts AYA-2004-03250, AYA2005-07808-C03-01, PNE2006-02934. European FEDER funds for these contracts are also acknowledged. PJG acknowledges financial support from the MEC (contract Ramon y Cajal).

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